Comment on ''Eigenfunction expansion of the dyadic Green's function in a gyroelectric chiral medium by cylindrical vector wave functions''

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We have examined the paper by Dajun Cheng [Phys. Rev. E **55**, 1950 (1997)]. It turns out that the theoretical results derived in the above paper are erroneous, and the method developed in the above paper is also questionable. [S1063-651X(99)08503-7]

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In a paper that appeared in this journal [1], Dajun Cheng claimed that, based on the Ohm-Rayleigh method, the dyadic Green's function in an unbounded gyroelectric chiral medium is rigorously represented in an eigenfunction expansion of the cylindrical vector wave functions. With considerable interest, we read the above paper carefully. Unfortunately, from Eqs. (19), (23), and (24)–(26) of Cheng's paper, it is evident that the dyadic Green's function derived by him does not satisfy the governing equation of the dyadic Green's function in a gyroelectric chiral medium, namely, Eq. (4) of Cheng's paper [1]. In other words, the electric field calculated from the dyadic Green's function, as shown in Eq. (3) of Cheng's paper is incorrect. Since Eq. (3) of Cheng's paper [1] is well known and correct, there is only one possibility: the dyadic Green's function derived by Cheng based on his new method is erroneous.

Giving an inverse example, which proves someone's result is wrong, is the best way to argue that a general expression is erroneous and a new method is questionable. In order to prove the above statement that the electric field calculated from the dyadic Green's function as shown in Eqs. (3), (19), (23), and (24)–(26) of Cheng's paper [1] is incorrect, we only need to check if the electric field outside the source region is correct or not. This can be easily checked by inspecting the electric field outside the source region of a dipole source, which is derived by the dyadic Green's function in a gyroelectric chiral medium. Actually, as stated by Cheng in the last paragraph of Sec. IV B [1], for a dipole source parallel to the z axis, only $\mathbf{V}_0^{(1)'}$ and $\mathbf{W}_0^{(1)'}$ terms exist for the dyadic Green's function. This means that, according to Cheng's formalism [see, for example, Eqs. (24)–(26) of Cheng's paper], outside the source region only $\mathbf{V}_0^{(1)}$ and $\mathbf{W}_0^{(1)}$ terms survive in the electric field representation. However, $\mathbf{V}_0^{(1)}$ and $\mathbf{W}_0^{(1)}$, including their linear combination, do not satisfy the corresponding homogeneous equation of Eq. (2) of Cheng's paper [1], i.e.,

$$\nabla\times\nabla\times\mathbf{E}-2\omega\mu\xi_c\nabla\times\mathbf{E}+\omega^2\mu\underline{\epsilon}\cdot\mathbf{E}=0. \tag{1}$$

From Eq. (1), we can see that

$$\nabla\cdot\mathbf{E}\neq 0 \tag{2}$$

if $\underline{\epsilon}$ is not a scalar. Before further discussion, for the sake of clarity, we briefly review the mathematical structure of solutions to Eq. (1) of this comment. If $\underline{\epsilon}$ is scalar and $\xi_c=0$, Eq. (1) can be separately satisfied by **L**, **M**, and **N**. The dynamic and propagating modes are **M** and **N**. If $\underline{\epsilon}$ is scalar and if $\xi_c\neq 0$, the dynamic mode solutions to Eq. (1) can be obtained in terms of the linear combination of **M** and **N**, namely, **V** and **W**. If $\underline{\epsilon}$ is a transversely isotropic tensor, as assumed in Cheng's papers [1,2], and if $\xi_c\neq 0$, the dynamic mode can be constructed by a finite sum of the special linear combination of **L**, **M**, and **N**, or, equivalently, a finite sum of the special linear combination of **L**, **V**, and **W**. If $\underline{\epsilon}$ is a general tensor, the dynamic mode can be constructed by an infinite sum of the special linear combination of **L**, **M**, and **N** [7,8]. In Ref. [1], the divergence of electric field is not zero, while the divergence of $\mathbf{V}_0^{(1)}$ and $\mathbf{W}_0^{(1)}$ is identically zero. In fact, as shown in Cheng's paper [2], it is not **L** and the linear combination of **V** and **W** separately that satisfy the corresponding homogeneous equation (2) of Cheng's paper [1] [our Eq. (1)], but a special linear combination of **L**, **V**, and **W** that satisfy the homogeneous wave equation in a gyroelectric chiral medium. Therefore [2], the linear combination of **V** and **W** cannot be a solution of Eq. (1). The correct solution of an electric field in a sourceless region must include the vector wave function **L**. It is well known that **V** and **W** satisfy the following standard wave equations of wave number k:

$$\nabla^2\mathbf{V}+k^2\mathbf{V}=0, \tag{3}$$

$$\nabla^2\mathbf{W}+k^2\mathbf{W}=0, \tag{4}$$

$$\nabla\times\mathbf{V}=k\mathbf{V}, \tag{5}$$

$$\nabla\times\mathbf{W}=-k\mathbf{W}, \tag{6}$$

$$\nabla\cdot\mathbf{V}=0, \tag{7}$$

$$\nabla\cdot\mathbf{W}=0. \tag{8}$$

Nobody would think that the standard wave equations (3) and (4) were the same as the complicated wave equation (1) in gyroelectric chiral medium. It is also easier work to perform a direct verification. As a matter of fact, when we substitute **V** or **W** or a linear combination of **V** and **W** into Eq. (1), the first two terms can cancel the third tensor term if and only if ϵ is a scalar. That means the homogeneous equation

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(1) is not satisfied if ϵ is a tensor. In other words, **V** or **W** or a linear combination of **V** and **W** is not a solution of the corresponding homogeneous equation of Eq. (2) in Ref. [1] [our Eq. (1)]. Cheng did not claim that **V** or **W** satisfies (1) either. Similarly, the dyadic Green's function of dipole sources perpendicular to the z axis contains only $\mathbf{V}_1^{(1)'}$ and $\mathbf{W}_1^{(1)'}$ terms [1]. According to Cheng's Eqs. (24)–(26), this means that outside the source region only $\mathbf{V}_{-1}^{(1)}$ and $\mathbf{W}_{-1}^{(1)}$ terms survive in the electric field representation. But $\mathbf{V}_{-1}^{(1)}$ and $\mathbf{W}_{-1}^{(1)}$, including their linear combination, do not satisfy [2] the corresponding homogeneous equation (2) of Cheng's paper [our Eq. (1)] either [1]. This point is also very evident from Eq. (2) of Ref. [1], which clearly shows that the divergence of electric field is not zero while the divergence of $\mathbf{V}_{-1}^{(1)}$ and $\mathbf{W}_{-1}^{(1)}$ is identically zero. Therefore the electric field derived by Cheng's dyadic Green's function does not satisfy the Maxwell equation, and thus is definitely incorrect. As a matter of fact, an exact solution to the wave equation of an electric field in a gyroelectric chiral medium must include vector wave functions of **L** kind as well as vector wave functions of **M** and **N** kinds [2]. In summary, by the dipole examples, we have proved that Cheng's electric field expression and therefore his dyadic Green's function are incorrect.

It should be emphasized that we do not misunderstand Cheng's paper [1]. Following Ref. [1], it is very evident that the full eigenfunction expansion of the dyadic Green's function is the summation of $\underline{\Gamma(\mathbf{r},\mathbf{r}')}$, $\underline{P}$, and $\underline{Q}$. In other words, Eqs. (19) and (13) or (24a) and (24b) of Ref. [1], together with Eq. (18), satisfy the governing equation (4). So it seems that the full eigenfunction expansion of the dyadic Green's function has included contributions of an irrotational vector function of **L** kind. However, if the author had not written Sec. III of Ref. [1], his result may have misled the reader to believe that his solution in some way had included a vector wave function of **L** kind in the dynamic or propagating modes. But as proven in Sec. III of Ref. [1], the introduction of a vector wave function of **L** kind only results in a static or local mode. Meanwhile, the tensor weights $\underline{P}$ and $\underline{Q}$ only determine the coefficients of the vector wave functions **M** and **N** or linear combinations of **M** and **N**, as clearly shown by Eqs. (9), (11), (13), and (14).

The reason why incorrect results are derived by Cheng is that the starting point, Eq. (9) of his paper, is questionable. It is not an eigenfunction expansion or Ohm-Rayleigh method [3] of the dyadic Green's function in a gyroelectric chiral medium, since the vector wave functions of **L**, **M**, and **N** are not eigenfunctions of the corresponding homogeneous wave equation in a gyroelectric chiral medium, although they are eigenfunctions of isotropic media. So a solution derived from Eq. (9) of Ref. [1] is a formal solution only. After getting the formal solution, a direct verification is required. A theoretical verification for isotropic media cannot guarantee the correctness on anisotropic media. Furthermore, the convergence of series (9) of Ref. [1] should be addressed.

According to Ohm-Rayleigh method [3] of the dyadic Green's function, both the unit dyadic δ source and the dyadic Green's function in a gyroelectric chiral medium should be expanded in terms of the eigenmodes of a given adjoint Maxwell system [2,4]. Instead of the simple orthogonality relationships of vector wave functions in isotropic media [1], a modal biorthogonality of given and adjoint eigenmodes must be applied [4,5]. Following the sophisticated procedure [4,5] above, the vector wave function of **L** can survive in the dynamic mode of an electric field [9], especially in the electric field of a dipole [9].

Finally, we would like to point out that following Cheng's method (Sec. III of his paper), the introduction of vector wave functions of **L** kind can only result in a local mode, or a static mode of the third kind [5,6]. Meanwhile the propagation modes, or the dynamic modes of the first and second kinds [2,6,7], include vector wave functions of **M** and **N** kinds only. The key point is that in a gyroelectric chiral medium the propagation modes, or the dynamic modes of the first and second kinds [2,5,6], must include both vector wave functions of **M** and **N** kinds and vector wave functions of **L** kind due to the nonzero divergence requirement of the electric field, which is a basic concept of vector wave function theory of anisotropic media [7,8]. While the completeness properties of the eigenfunction set of the gyroelectric chiral medium have been demonstrated [5,9] Cheng's theoretical results cannot be correct. Actually, the classification of three kinds of modes, namely, one kind of static mode and two kinds of dynamic modes, can be found in more popular literature such as Refs. [10–12].

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